FCS-MPC and Observer Design for a VSI with Output LC Filter and Sinusoidal Output Currents

Sergio Vazquez, Abraham Marquez, Jose I. Leon and Leopoldo G. Franquelo
Electronic Engineering Department
Universidad de Sevilla
Sevilla, Spain
Email: sergi@us.es

Tobias Geyer
ABB Corporate Research
5405 Baden-Dättwil, Switzerland
Email: t.geyer@ieee.org

Abstract—**Voltage source inverter (VSI) with output LC filter can be used to generate sinusoidal output voltages with reduced low frequency harmonics content. This application is suitable for uninterruptible power supply (UPS) systems. Finite control set model predictive control (FCS-MPC) has proved to be a good candidate for controlling such kind of devices. FCS-MPC relies on accurate system model to achieve high performance. However, output load is not always known for UPS applications. Under this condition, the use of observers to estimate the output load currents is a good solution. In this paper, a FCS-MPC strategy using an unknown input observer (UIO) is assessed. To design the UIO, the nature of the output load has been considered. This paper is focused on output loads with sinusoidal output currents. Two different UIO are evaluated. The first one uses a conventional approach and the second one takes into account the sinusoidal nature of the output load currents. Experimental results in a VSI prototype show that the second approach can provide superior performance independently of the output load connected to the power inverter.**

I. Introduction

Applications for voltage source inverters (VSI) with output LC filter have a great importance in fields like renewable energy, high performance drives, distributed generation, energy storage systems, etc [1], [2]. These kind of power converters have especial interest for uninterruptible power supply (UPS) or supplying critical loads [3]. In these applications, the main objective is to provide a sinusoidal output voltage free of low order harmonic content no matters the output load value.

Several control strategies have been proposed for this system. For instance, resonant, repetitive, robust or predictive controllers [4]–[7]. This paper is focused on model predictive control (MPC) applied to VSI with output LC filter [8]. In particular a finite control set MPC (FCS-MPC) is studied [9]. This control family is very attractive because it provides a very fast dynamic response [10].

FCS-MPC takes advantage of the discrete nature of the VSI. The basic idea is to use a model of the system to calculate predictions, up to certain prediction horizon, for all the available switching vectors. Then a cost function is minimized and the voltage vector that minimizes this cost function is applied the next sample step. The whole process is repeated each sample step in a receding horizon fashion. Additionally, FCS-MPC allows one to include nonlinearities in the cost function [11], [12]. In this paper, the FCS-MPC strategy for VSI with output LC filter presented in [13] is adopted. This strategy uses a prediction horizon of one, but it is possible to use a larger prediction horizon value [14], [15].

In [13] the proposed FCS-MPC uses an observer in order to estimate the output load currents. This is of great interest because it allows one to reduce the system cost avoiding the use of extra current sensors to measure these values. Several works can be found facing this issue for this kind of system [16], [17]. A generic observer is designed in [13] without introducing any knowledge about the output load. Although this provides good results, more advantages can be found when the nature of the load is considered in the observer design process. This paper is focused on output loads drawing sinusoidal output currents from the inverter. When this is considered, it is possible to design a more specific observer.

The remainder of this paper is organized as follows. In Section II the FCS-MPC is described. In Section III both observer designs are addressed, firstly the one proposed in [13] and secondly the new approach. Experimental results are documented in Section IV, where steady state and transient response are analyzed. Finally, conclusions are given in Section V.

II. FCS-MPC for Inverter with output LC filter

The FCS-MPC strategy for an inverter with output LC filter for UPS application was presented in [13]. Here a brief summary of the algorithm is presented.

A. Model of the system

The electric circuit of a two level voltage source inverter (VSI) with output LC filter connected to a load is depicted in Fig. 1. Here copper losses have been considered negligible in the output LC filter. The system variables and parameters are described in Table I.

Using the Clarke's transformation, the dynamic model in the stationary $\alpha\beta$ frame of the output LC filter can be expressed as:

$$v_{I,\alpha\beta} = L\frac{di_{L,\alpha\beta}}{dt} + v_{C,\alpha\beta} \tag{1}$$

$$i_{L,\alpha\beta} = C\frac{dv_{C,\alpha\beta}}{dt} + i_{O,\alpha\beta}. \tag{2}$$

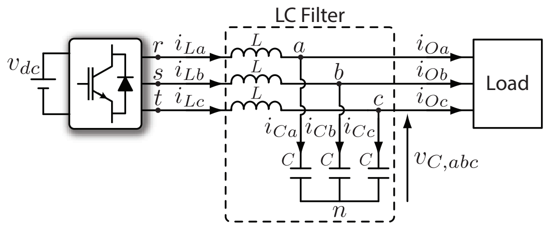

Fig. 1. Scheme of the inverter connected to the output LC filter and load.

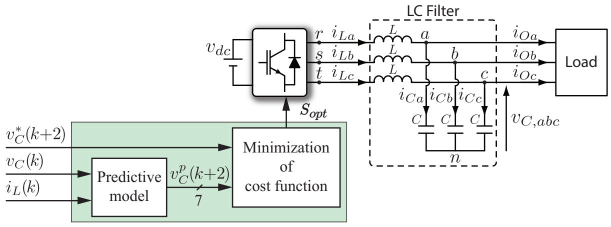

Fig. 2. Block diagram of the FCS-MPC applied to the VSI with output LC filter.

TABLE I
SYSTEM VARIABLES AND PARAMETERS

Variable	Description
$v_{C,abc} = \{v_{an}\, v_{bn}\, v_{cn}\}^T$	Output filter capacitor voltage vector
$i_{L,abc} = \{i_{La}\, i_{Lb}\, i_{Lc}\}^T$	Output filter inductor current vector
$v_{I,abc} = \{v_{rn}\, v_{sn}\, v_{tn}\}^T$	VSI output voltage vector
$i_{O,abc} = \{i_{Oa}\, i_{Ob}\, i_{Oc}\}^T$	Output load current vector
L	Output filter inductance
C	Output filter capacitance
v_{dc}	dc-link voltage

A state space model can be derived form (1) and (2). Then, it can be transformed to the discrete-time domain. This discrete-time state space model is used for calculating predictions of the output filter capacitor voltage and inductor current. The discrete-time model of the system for a sampling time T_s, is written as [13]:

$$x(k+1) = A_q x(k) + B_q v_{I,\alpha\beta}(k) + B_{dq} i_{O,\alpha\beta}(k), \quad (3)$$

where state variable x is

$$x = \left[i_{L,\alpha\beta}^T \; v_{C,\alpha\beta}^T\right]^T \quad (4)$$

and matrices A_q, B_q and B_{dq} are

$$A_q = e^{AT_s};\; B_q = \int_0^{T_s} e^{A\tau} B d\tau;\; B_{dq} = \int_0^{T_s} e^{A\tau} B_d d\tau, \quad (5)$$

being

$$A = \begin{bmatrix} 0 & -1/L \\ 1/C & 0 \end{bmatrix},\; B = \begin{bmatrix} 1/L \\ 0 \end{bmatrix},\; B_d = \begin{bmatrix} 0 \\ -1/C \end{bmatrix}. \quad (6)$$

B. FCS-MPC algorithm

The FCS-MPC algorithm for the VSI with output LC filter is applied each sampling instant k. It consists on choosing among the available switching states, the one that minimizes the cost function

$$g = \|v_{C,\alpha\beta}^*(k{+}2) - v_{C,\alpha\beta}^p(k{+}2)\|_2^2 \quad (7)$$

The selected switching state, S_{opt}, is considered as the optimal one and it is applied to the VSI the next sampling instant $k+1$. This procedure is repeated each sampling period. FCS-MPC takes advantage of the discrete nature of the VSI. Thus when the problem is formulated in the stationary $\alpha\beta$ frame the number of available switching states for the two level power converter is 7.

To evaluate the cost function (7) the output capacitor voltage reference $v_{C,\alpha\beta}^*$ and its prediction $v_{C,\alpha\beta}^p$ are needed. It should be noticed that both values are evaluated at instant $k+2$ in order to handle with delay compensation issues [18]. Fig. 2 shows the block diagram of the FCS-MPC applied to the VSI with output LC filter.

III. OBSERVER DESIGN

Predictions $v_{C,\alpha\beta}^p$ are calculated using the predictive model represented by (3). It should be noticed that for evaluating (3) it is necessary to know the output load current values. These currents can be measured, but this increases the cost of the system. Therefore $i_{O,\alpha\beta}$ should be estimated in order to avoid the use of the extra sensors. In this way, an observer is used to estimate these currents.

The observer design can be done using several techniques. In this work, the Unknown Input Observer (UIO) is adopted to solve the problem [19]. This approach can provide good results, but it needs to make assumptions about the load model. Two options will be compared. On one hand, it will be considered constant output current between sampling instants as the load model [13]. On the other hand, a sinusoidal output current between sampling instants for the load model is proposed. It should be noted, that this new approach fits with the sinusoidal output load nature.

A. Constant output current between samples

To develop the UIO, the use of a constant output current between sampling instants as the load model was proposed in [13]. Under this assumption:

$$\frac{di_{O,\alpha\beta}}{dt} = 0. \quad (8)$$

Including this load model in the system, the state space model for the observer design yields:

$$\frac{dx_{obs}}{dt} = A_{obs} x_{obs} + B_{obs} v_{I,\alpha\beta}, \quad (9)$$

where

$$x_{obs} = \left[i_{L,\alpha\beta} \; v_{C,\alpha\beta} \; i_{O,\alpha\beta}\right]^T \quad (10)$$

and

$$y_{obs} = C_{obs} x_{obs}, \quad (11)$$

where y_{obs} is composed of the measurements of $i_{L,\alpha\beta}$ and $v_{C,\alpha\beta}$. Then, the UIO can be defined as

$$\frac{d\hat{x}_{obs}}{dt} = A_{obs}\hat{x}_{obs} + B_{obs}v_{I,\alpha\beta} + M\left(y_{obs} - \hat{y}_{obs}.\right), \quad (12)$$

where $\hat{x}_{obs}$ and $\hat{y}_{obs}$ are the estimates for x_{obs} and y_{obs}, respectively. The output for the UIO system is $\hat{y}_{obs} = C_{obs}\hat{x}_{obs}$ and M is the so called observer matrix gain [20]. Expressions for matrices A_{obs}, B_{obs} and C_{obs} can be found in [13].

B. *Sinusoidal output current between samples*

In this paper, a new proposal for the load model is done. Taking into account that the load connected to the output LC filter presents sinusoidal output currents, then the output current can be expressed as

$$i^{T}_{O,\alpha\beta} = I_o[cos(\omega t + \phi)\ sin(\omega t + \phi)], \quad (13)$$

where ω is the output voltage frequency, I_o is the output load current amplitude and ϕ is the angle between $v_{C,\alpha\beta}$ and $i_{O,\alpha\beta}$, which depends on the power factor of the load. The derivative of (13) over the time is

$$\frac{di_{O,\alpha\beta}}{dt} = J\omega i_{O,\alpha\beta}; J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad (14)$$

that is proposed as the new load model for the UIO design.

Using (14) as the load model yields the state space model for the observer (9). But now matrices A_{obs} and B_{obs} are defined as:

$$A_{obs} = \begin{bmatrix} 0 & -1/L & 0 \\ 1/C & 0 & -1/C \\ 0 & 0 & J\omega \end{bmatrix},\ B_{obs} = \begin{bmatrix} 1/L \\ 0 \\ 0 \end{bmatrix}. \quad (15)$$

Additionally, the output of the system is defined as (11) with

$$C_{obs} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}. \quad (16)$$

Now that new matrices A_{obs}, B_{obs} and C_{obs} have been determined, the UIO can be designed as previously in (12). The new observer also uses the estimates for the state variables and system outputs $\hat{x}_{obs}$ and $\hat{y}_{obs}$, respectively. The last one is calculated by $\hat{y}_{obs} = C_{obs}\hat{x}_{obs}$. It should be noticed that gain matrix M also appears in the observer design.

IV. Experimental results

The FCS-MPC controller with the proposed observer was tested on a VSI with output LC filter prototype. The three-phase load consists of a resistor and inductor connected in series per phase. The voltage reference is a three-phase sinusoidal signal of 230 V_{rms} and 50 Hz. The algorithms are implemented on a DSPACE 1103 platform. Table II summarizes the system parameters.

Two set of experiments have been conducted. On one hand, the steady state behavior has been evaluated. For this purpose a fixed load has been connected to the VSI output. This load consists of a resistor of 60 Ω and an inductor of 20 mH connected in series per phase. This stands for a total load of 2.63 kVA. On the other hand, the transient response of the controller has also been assessed. To do this, a load step has been produced. The load changes from the previous one to a resistor of 15 Ω and an inductor of 20 mH connected in series per phase. This means a load step from 2.63 kVA to 9.76 kVA.

TABLE II
System parameters

Parameter	Value
DC-Link voltage v_{dc}	700 V
Output filter inductance L	2 mH
Output filter capacitance C	50 μF
Output load 0: R_0, L_0	60 Ω, 20 mH
Output load 1: R_1, L_1	15 Ω, 20 mH
Sampling period	40 μs

TABLE III
Steady state results

	Observer in [13]	Proposed observer
J_{gain}	1e4	1e4
THD (%)	1.80	1.74

A. *Steady state analysis*

The performance of the FCS-MPC controller and observer should be studied to assess the performance of the overall system. Both responses have been evaluated when observers based on constant and sinusoidal output current between samples are used.

The system behavior depends on the observer gain matrix M. This matrix should be chosen such that the poles of the observer give dynamics which are several times faster than the open-loop system dynamics [13]. In this work the poles of the observers have been chosen as

$$P_{obs} = J_{gain}\begin{bmatrix} -1 - 0.1j & -1 + 0.1j & -0.1 \end{bmatrix}, \quad (17)$$

where J_{gain} represents a design parameter. Tuning this value should take into account the tradeoff between bandwidth and noise rejection.

Fig. 3, Fig. 4 and Fig. 5 show the performance of the FCS-MPC and the observer proposed in [13] when $J_{gain} = 1e4$. It is clear that the controller is able to track the voltage reference but a steady state low frequency error appears. Regarding the observer, output current estimate does not match with the actual current value. However, the observer presents a good noise rejection due to the chosen value of J_{gain}. To increase J_{gain} improves the tracking of the actual current but high frequency noise appears diminishing the system performance.

The new proposed observed is tested using the same value for J_{gain}. Fig. 6, Fig. 7 and Fig. 8 display the steady state results. The proposed observer with this gain value provides an output current estimate that matches the actual current and achieves a good noise rejection. This improves the performance of the FCS-MPC that tracks the voltage reference with smaller low frequency error and lower high frequency ripple compared with counterpart observer proposed in [13].

Table III summarizes the THD values of the output voltage obtained in steady state with the different observers. The THD value is calculated up to harmonic 250th. The results show that the proposed observer can achieve better performance with the same gain compared with the observer proposed in [13].

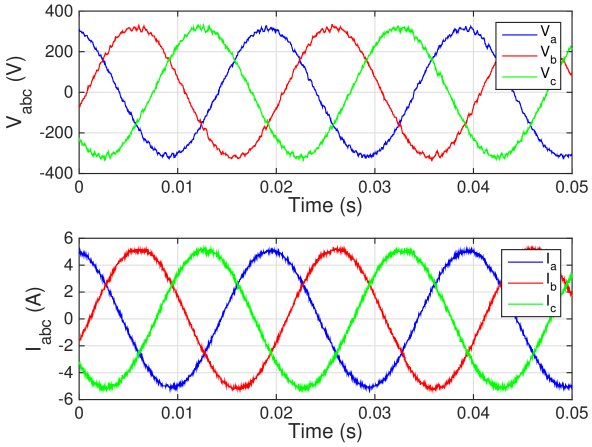

Fig. 3. Performance of the FCS-MPC controller with observer proposed in [13] and $J_{gain} = 1e4$. Top: Output voltages. Bottom: Output currents.

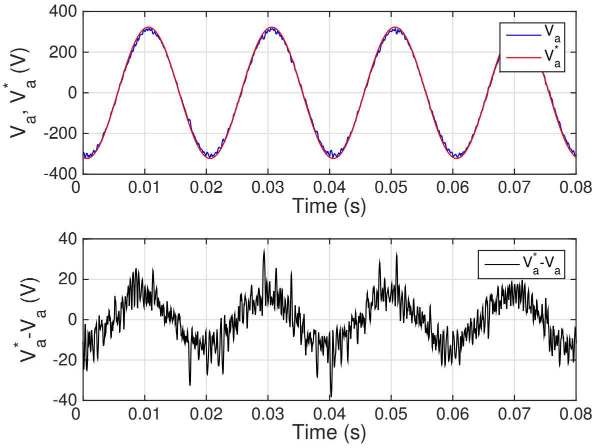

Fig. 4. Performance of the FCS-MPC controller with observer proposed in [13] and $J_{gain} = 1e4$. Top: Output voltage and reference for phase a. Bottom: Tracking error for phase a.

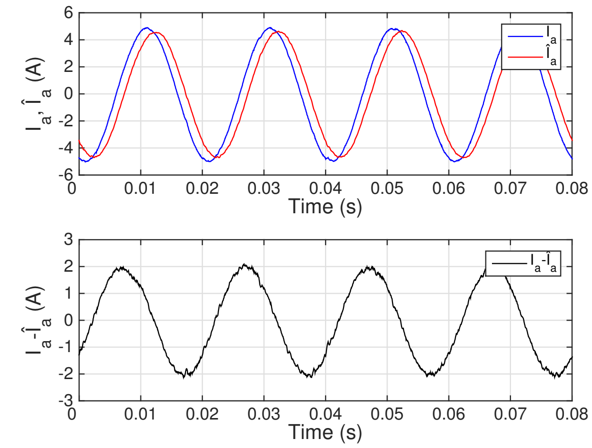

Fig. 5. Performance of the FCS-MPC controller with observer proposed in [13] and $J_{gain} = 1e4$. Top: Output current and estimate for phase a. Bottom: Difference between actual and estimated current for phase a.

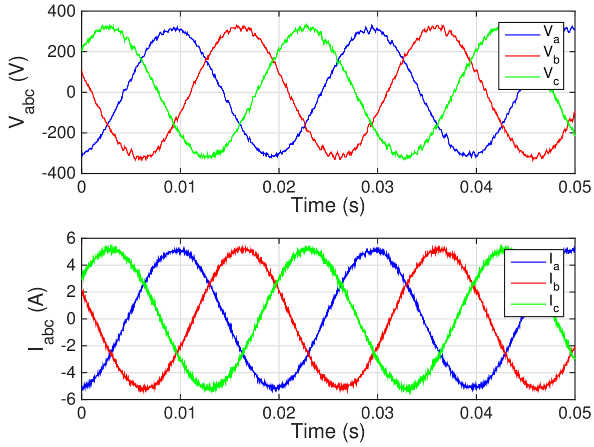

Fig. 6. Performance of the FCS-MPC controller with the new proposed observer and $J_{gain} = 1e4$. Top: Output voltages. Bottom: Output currents.

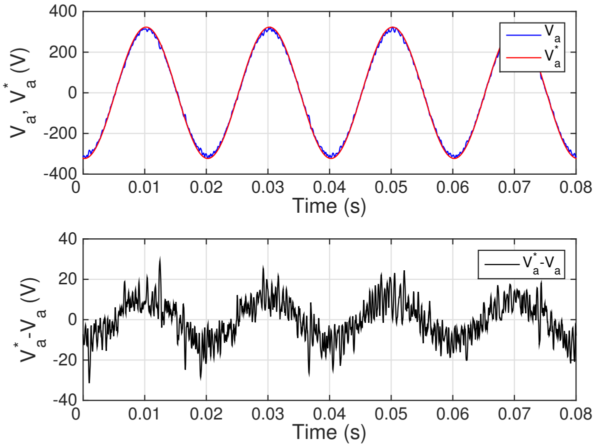

Fig. 7. Performance of the FCS-MPC controller with the new proposed observer and $J_{gain} = 1e4$. Top: Output voltage and reference for phase a. Bottom: Tracking error for phase a.

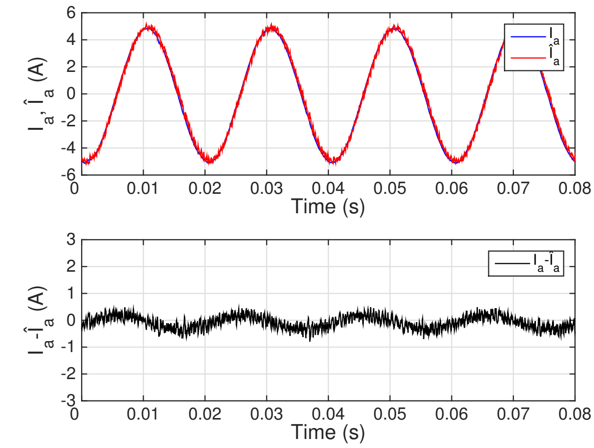

Fig. 8. Performance of the FCS-MPC controller with the new proposed observer and $J_{gain} = 1e4$. Top: Output current and estimate for phase a. Bottom: Difference between actual and estimated current for phase a.

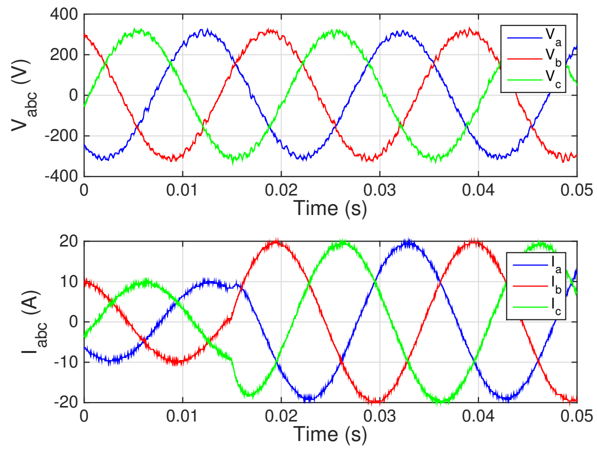

Fig. 9. Performance of the FCS-MPC controller with observer proposed in [13] and $J_{gain} = 1e4$ under load step. Top: Output voltages. Bottom: Output currents.

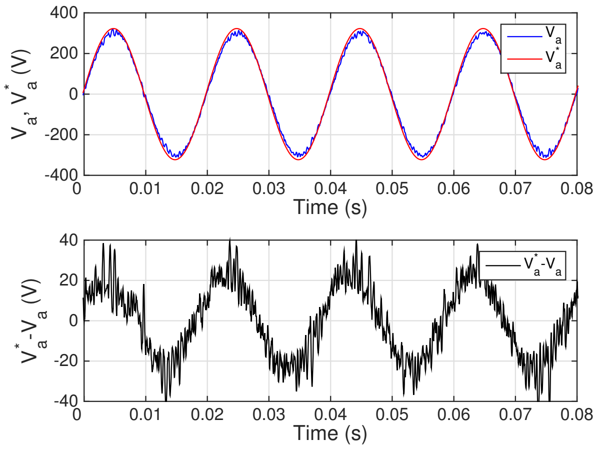

Fig. 10. Performance of the FCS-MPC controller with observer proposed in [13] and $J_{gain} = 1e4$ under load step. Top: Output voltage and reference for phase a. Bottom: Tracking error for phase a.

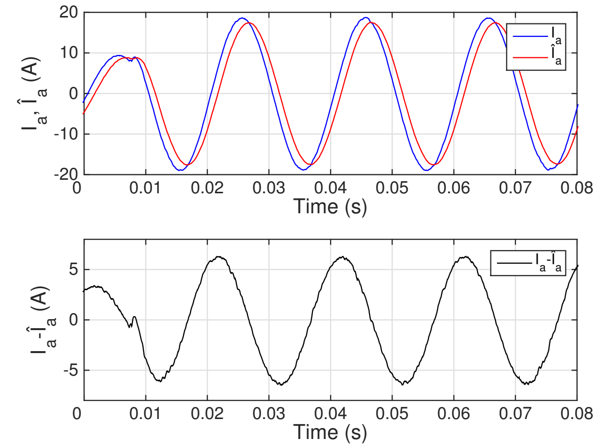

Fig. 11. Performance of the FCS-MPC controller with observer proposed in [13] and $J_{gain} = 1e4$ under load step. Top: Output current and estimate for phase a. Bottom: Difference between actual and estimated current for phase a.

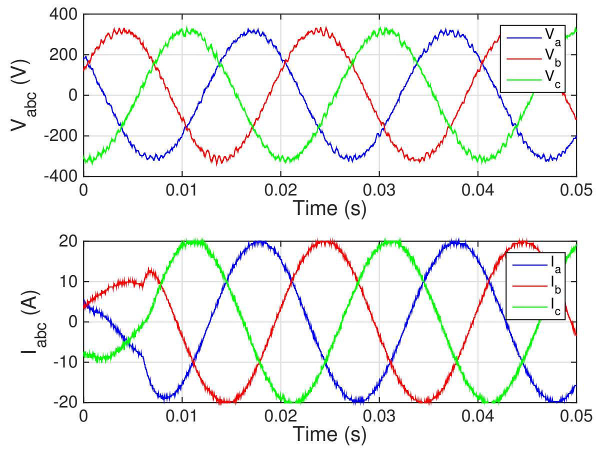

Fig. 12. Performance of the FCS-MPC controller with the new proposed observer and $J_{gain} = 1e4$ under load step. Top: Output voltages. Bottom: Output currents.

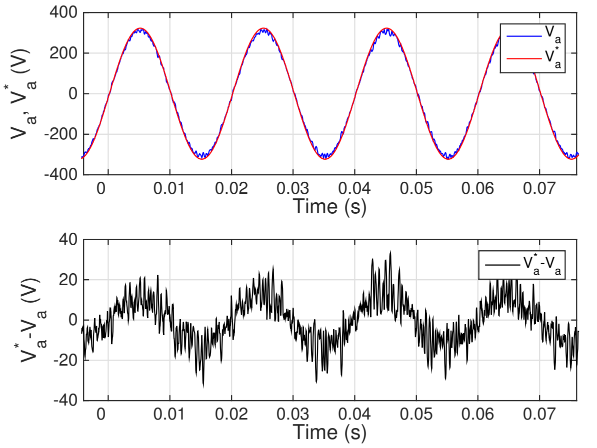

Fig. 13. Performance of the FCS-MPC controller with the new proposed observer and $J_{gain} = 1e4$ under load step. Top: Output voltage and reference for phase a. Bottom: Tracking error for phase a.

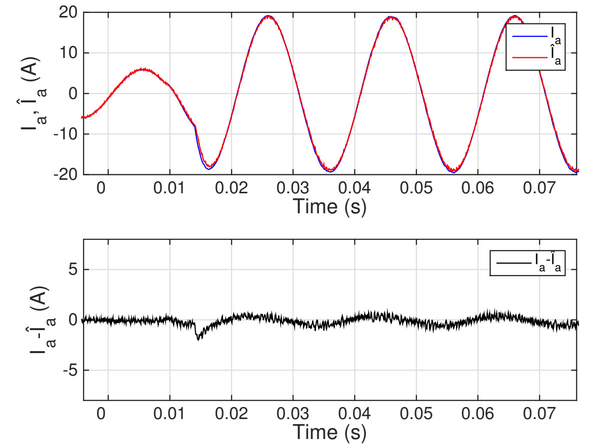

Fig. 14. Performance of the FCS-MPC controller with the new proposed observer and $J_{gain} = 1e4$ under load step. Top: Output current and estimate for phase a. Bottom: Difference between actual and estimated current for phase a.

B. Transient response analysis

Similar to steady state, the performance of the FCS-MPC and the observer should be evaluated during a transient response for assessing the performance of the overall system.

As in Section IV-A, $J_{gain} = 1e4$ has been considered for the study. Fig. 9, Fig. 10 and Fig. 11 show the results when the observer proposed in [13] is used. It is clear that the controller performance depends on the load value. Although the voltage tracks its reference, the low frequency error increases as the actual load differs from its model. The high frequency ripple is also present, but it seems to be independent from the load value. The main reason for the controller performance to get worse is that output current estimate gets away from the actual current value. Therefore, it is possible to tune the design parameter to achieve good behavior for a certain load value, but once it changes, the system behavior deteriorates.

The new proposed observer is evaluated in Fig. 12, Fig. 13 and Fig. 14. The system behaves correctly even under a load step conditions. The low frequency error level and the high frequency ripple are maintained independently of the connected load. The main reason for that is that the new proposed observer can estimate better the output current compared to the observer in [13]. This is because the load model is more accurate and it fits better to the actual load than the one proposed in [13]. Thus it is possible to tune the design constant for a certain load and expect to achieve the same system performance to any other load value. This represents a clear advantage compared to previous observer proposal, as system behavior can be maintained independently of the output load.

V. Conclusion

Finite control set model predictive control (FCS-MPC) is a good alternative for controlling voltage source inverters (VSI) with output LC filter. However, FCS-MPC needs a good model of the system for achieving high performance. In uninterruptible power supplies, connected load characteristics are not always known. Therefore, the use of an observer allows one to estimate the output load currents without extra current sensors. In this paper, loads drawing sinusoidal output currents from the inverter are considered. Then, two observer designs are compared. The first one was proposed previously and it does not make any assumption about the load nature. The second one is a new proposal and takes into account the expected behavior of the load currents. Practical results showed that the new proposed observer maintains system performance independently of the output load. On the contrary, the previous solution fails in this objective. Therefore, the new approach is preferable for large range of output load values.

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