HYBRID EMERGENCY VOLTAGE CONTROL
IN POWER SYSTEMS

Tobias Geyer*, **Mats Larsson**†, **Manfred Morari***

* Automatic Control Laboratory, ETH Zentrum – ETL, CH-8092 Zürich, Switzerland
tel: +41-1-632 7626, fax: +41-1-632 1211, {geyer,morari}@aut.ee.ethz.ch

† Corporate Research, ABB Switzerland, CH-5405 Baden-Dättwil, Switzerland
tel: +41-58-586 8292, fax: +41-58-586 7365, mats.larsson@ch.abb.com

Abstract

This paper presents a novel emergency control scheme capable of predicting and preventing a voltage collapse in a power system, that is modelled as a hybrid system incorporating nonlinear dynamics, discrete events and discrete manipulated variables. Model Predictive Control in connection with the Mixed Logical Dynamical framework is used to successfully stabilize the voltage of a four bus example system.

1 Introduction

An electrical power system consists of numerous components connected together to form a large, complex system generating, transmitting and distributing electrical power. Electric power systems and additional preventive control schemes are designed in such a way, that the system should be able to withstand any single contingency, that is, outage of any single component without loss of stability and with all system variables kept within predefined ranges [10]. Not all possible disturbances, however, can be foreseen at the planning stage and these may result in instability leading eventually to collapse or islanding of the system. Furthermore, because of environmental constraints on the extension of the transmission capacity, increased electricity consumption and new economic constraints imposed by the liberalized power market, power systems are operated closer and closer to their stability limits.

Under heavy load conditions, a power system can become unstable exhibiting slow voltage drops, that may lead to a voltage collapse resulting in a black-out if appropriate countermeasures are not taken. Several such voltage instability incidents have occurred around the world [17, 20]. As a consequence, voltage stability has become a major concern in power system planning and operation and the need for emergency control schemes that ensure stability - also during cascaded or multiple outages - has increased.

Currently, most practical implementations of protection systems against voltage collapses are purely rule-based and most often based only on local criteria [6]. General rules are to disconnect load and to connect any available capacitor bank if the voltage drops to abnormally low levels. While local protection schemes have the obvious advantage of simplicity since they do not require wide-area communication, it has been been shown, that substantial benefits can be gained by coordinating the actions taken in different parts of the system [13]. In particular, proper coordination of all control measures minimizes the amount of load shed. Additionally, because of the nonlinearity of the power system, it is very difficult to specify a single appropriate rule for the complete range of operating conditions. Traditionally, these rules are tuned on an *ad-hoc* basis [6]. In [15] however, a *systematic* approach has been reported, where an optimally tuned rule is derived by solving a combinatorial optimization problem.

The nonlinear behavior of the system calls for methods based on a dynamic model in order to account for changes of the operating point and the network state. This is complicated by the fact, that most control moves are inherently discrete-valued. Examples include capacitor banks and tap changers, which must be switched using fixed step sizes, and load-shedding which must be carried out by disconnecting whole feeders since most utilities lack direct load control schemes. Recent advances in computation, communication and power system instrumentation technology, more specifically Phasor Measurement Units and Wide-Area Measurement Systems [16], have made coordinated and model based approaches tractable. They are highly attractive since the use of a model in combination with on-line optimization allows for optimal coordination of different control moves and automatic adaption to changing operating conditions. Thanks to this, they are less conservative than non-adaptive local schemes - even if the local schemes have been optimally tuned - and thus avoid unnecessary operation of the protection schemes in order to minimize load shedding.

Model Predictive Control (MPC) has been used successfully for a long time in the process industry and recently also for hybrid systems [3]. MPC was first applied to the emergency voltage control problem in [12]. There, the MPC problem was based on a *single linearized* model for the whole prediction horizon. The discrete-valued control variables led to a combinatorial optimization problem which was solved by a tree search. In [13], this work was extended by heuristic search enhancements in order to render the problem tractable also for large-scale applications.
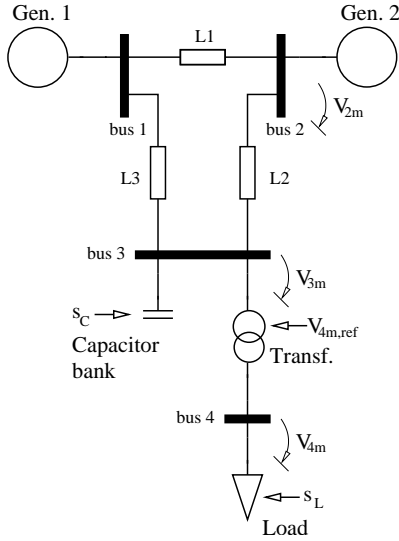
Figure 1: Example power system.

As detailed above, power systems are hybrid systems incorporating not only discrete manipulated variables but also discrete events like logic and finite state machines. Although there exist various different methodologies to model hybrid systems, their equivalence can be shown under mild assumptions [8]. In this paper, we will focus on the *Mixed Logic Dynamical (MLD)* framework [3]. In comparison to the previous work [12, 13], the MLD framework allows us to model the *hybrid* behavior of the power system and thus provides better accuracy since effectively *multiple linearizations* are used during the prediction interval. It is believed that this is the first time a general-purpose control framework is applied successfully to the emergency voltage control problem.

The paper is structured in the following way. Section 2 summarizes the model of the example power system and Section 3 details its formulation as MLD model. The optimal control problem is stated and solved in Section 4. Section 5 concludes the paper.

2 Example Power System

In this paper, we will focus on the power system given by [11, 7], which is contained in the EU project *Control and Computation (CC)* as a case study. This power system incorporates all the components of a mature power system and can be driven unstable exhibiting a voltage collapse. Nevertheless, it is small enough to serve as a starting point allowing for a successful derivation of an emergency control scheme.

2.1 Overview

As depicted in Figure 1, the power system contains two generators. The generator 1 is modelled as an infinite bus, i.e. a large power system, whereas generator 2 can only produce a limited amount of reactive power. The latter includes an internal controller regulating the voltage at bus 2. The two generators are

connected with each other and the transformer by three transmission lines. All these components form the transmission system of the power system.

The transformer incorporates an internal controller regulating the load voltage V_{4m} within a dead-band around the voltage reference $V_{4m,ref}$. This controller is a finite state machine and allows changes of the tap position n_T only every $30\,\mathrm{s}$ by one discrete step. In addition, by setting s_C, parts of the capacitor bank can be used to support the power system by producing reactive power close to the load.

The distribution system that, in general, consists of numerous loads on different voltage levels connected with each other by transformers, is modelled using one load model aggregating and approximating the whole distribution system. It is connected to the secondary side of the transformer. Discrete parts of the load can be disconnected by using s_L.

Summing up, the power system has one continuous constrained manipulated variable, namely the reference voltage of the tap changer $V_{4m,ref} \in [0.8, 1.2]$[1], and two integer manipulated variables given by the number of discrete capacitor banks connected to the system $s_C \in \{0, 1, 2, 3\}$ and the discrete amount of load-shedding $s_L \in \{0, 1, 2, 3\}$. The outputs of main interest are the three continuous bus voltages $V_{im} \in \mathbb{R}, i \in \{2, 3, 4\}$. Besides that, the power system has the following continuous and discrete-valued states. The two continuous states $x_{Lp}, x_{Lq} \in \mathbb{R}$ describe the dynamics of the active and reactive power in the load. The discrete-valued variable $n_T \in \{0.8, 0.82, 0.84, \ldots, 1.2\}$ denotes the tap position of the transformer. Additional binary states are needed to model the finite state machine of the internal tap changer controller.

As detailed in [7], the power system under consideration is a hybrid system containing integer manipulated variables, a saturation, a finite state machine with thresholds and boolean logic, two ordinary differential equations and 29 algebraic equality constraints of which 18 are nonlinear. The detailed mathematical model including the parameters is given in [7], but omitted here due to space limitations.

2.2 Decomposition

The power system which incorporates both continuous dynamics and discrete events can be divided accordingly into the following two parts:

(i) The continuous dynamical system. This part is modelled as a differential algebraic equation (DAE) system which is formed from the two ordinary differential equations (ODE) of the load and the algebraic equations (equality constraints). The ODE as well as most of the algebraic equations are nonlinear. Additionally, the saturation of the internal Automatic Voltage Regulator (AVR) of generator 2 can be included as part of the continuous dynamics. The discrete-valued n_T and the integers s_C and s_L form the inputs to this subsystem, the three bus voltages

---

[1]All equations, variables and parameters are normalized using the per-unit system (p.u.), as it is common practise in the power system community.
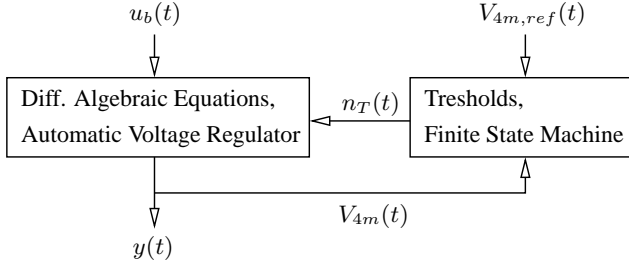
Figure 2: Model decomposition.

$V_{im}, i \in \{2, 3, 4\}$ constitute the outputs, and x_{Lp}, x_{Lq} are the continuous states.

(ii) The discrete event system. It combines the thresholds and the finite state machine modelling the internal controller of the transformer. $V_{4m,ref}$ and the bus voltage V_{4m} are the continuous inputs, n_T is both the discrete-valued output and the state.

The model decomposition is shown in Figure 2, where $x := [x_{Lp}, x_{Lq}]^T$, $u_b := [s_C, s_L]^T$ and $y := [V_{2m}, V_{3m}, V_{4m}]^T$.

For completeness, the initial values of the load states are given by $x_{Lp}(0) = x_{Lq}(0) = 0$, the position of the tap changer is $n_T(0) = 1.02$ and the steady state inputs to the nominal system are $V_{4m,ref} = 1$, $s_C = 2$ and $s_L = 0$.

2.3 Fault

At time $t = 100\,\mathrm{s}$ a short circuit occurs in the transmission line L3 which is immediately partly disconnected. As a consequence, the admittance of the line is reduced from 2 to $\frac{2}{3}$, which increases the power losses on the line and constricts the power transmitted from generator 1 to the load. Therefore, the second generator is put under heavier load. The dynamics of the load together with the internal controller of the tap changer drive the generator 2 to its loading limit. The overexcitation limiter of the second generator is activated at time $t = 224\,\mathrm{s}$ leading to a loss of voltage control and to a fast voltage collapse within less than 200 s as shown in Figure 3.
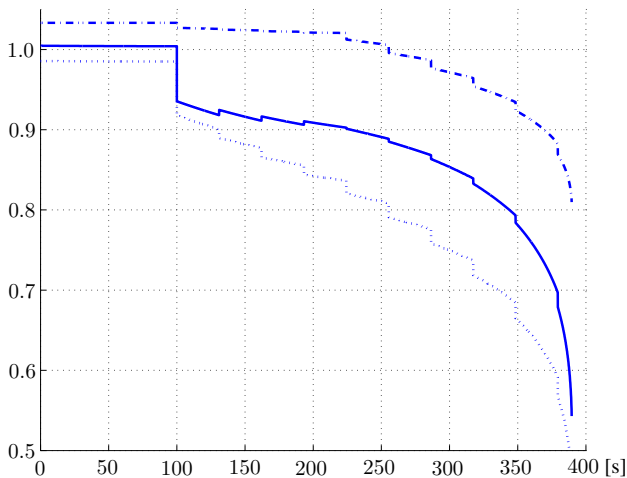

Figure 3: Bus voltages of open-loop response: V_{2m} is the dash-dotted, V_{3m} the dashed and V_{4m} is the solid line.

3 MLD Model

The general MLD form of a hybrid system introduced in [3] is

$$x(t+1) = Ax(t) + B_1 u(t) + B_2 \delta(t) + B_3 z(t) \qquad (1a)$$
$$y(t) = Cx(t) + D_1 u(t) + D_2 \delta(t) + D_3 z(t) \qquad (1b)$$
$$E_2 \delta(t) + E_3 z(t) \leq E_4 x(t) + E_1 u(t) + E_5, \qquad (1c)$$

where $x \in \mathbb{R}^{n_c} \times \{0,1\}^{n_\ell}$ denotes the states, $u \in \mathbb{R}^{m_c} \times \{0,1\}^{m_\ell}$ the inputs and $y \in \mathbb{R}^{p_c} \times \{0,1\}^{p_\ell}$ the outputs, with both continuous and binary components. Furthermore, $\delta \in \{0,1\}^{r_\ell}$ and $z \in \mathbb{R}^{r_c}$ represent binary and auxiliary continuous variables, respectively. These variables are introduced when translating propositional logic or PWA functions into linear inequalities. All constraints on state, input, and auxiliary variables are summarized in the inequality (1c). Note that the equations (1a) and (1b) are linear; the nonlinearity is hidden in the integrality constraints over the binary variables.

We consider MLD systems that are *completely well-posed* [3], i.e. for given $x(t)$ and $u(t)$, the values of $\delta(t)$ and $z(t)$ are uniquely defined by the inequality (1c). This assumption is not restrictive and is always satisfied when real plants are described in the MLD form [3].

Based on Section 2.2, the nonlinear model of the power system can be cast into MLD form using two different approaches.

The first approach is to approximate the continuous-time dynamics of the DAE by discrete-time dynamics and subsequently to approximate the nonlinearities of the discrete-time dynamics and the algebraic equations by PWA functions. This can been done using an algorithm based on [5] exploiting the combined use of clustering, linear identification and classification techniques allowing us to identify at the same time both the affine functions and the polyhedral partition of the domain on which each affine function is valid. However, as most of the involved expressions are strongly nonlinear and defined over two- or three-dimensional domains and because the interactions between the nonlinear algebraic constraints are tight, this approach leads to a disproportionally large number of polyhedra and unacceptable large approximation errors [7].

On the other hand, approximating the input to output behavior of the entire DAE system with the AVR reduces not only the number of polyhedra but also leads to significantly smaller approximation errors. This subsystem has the continuous states x_{Lp} and x_{Lq}, the discrete-valued input n_T and the integer inputs s_C and s_L. Gridding the state-input space spanned by x_{Lp}, x_{Lq} and n_T, and sampling the system response for each combination of integer inputs yields the discrete-time state-update functions of the load as well as the output functions of the bus voltages.

The number of states can be reduced by one by observing that the ratio between the load states is almost constant, i.e. $\frac{x_{Lp}}{x_{Lq}} \in [9.995, 10.4]$. The function $r(V_{4m}) := 10 + 4.75(\frac{1}{V_{4m}} - 1)$ approximates $\frac{x_{Lp}}{x_{Lq}}$ well for $x_{Lp} \in [-1, 10]$ observed during experiments. Due to the fact that the DAE with the AVR is

only mildly nonlinear, it is sufficient to partition the resulting two-dimensional state-input space, which is spanned by x_{Lp} and n_T, into 24 triangular polytopes or simplices. Then, the toolbox [9] yields the PWA state-update and output functions for a given integer input combination. As $s_C = 0$ or $s_C = 1$ would destabilize the system and $s_L > 1$ is never necessary in order to stabilize the system, we only consider $s_C = \{2, 3\}$ and $s_L = \{0, 1\}$. This helps to reduce the complexity of the MLD model. Note that the partition does not depend on the integer inputs. For each binary input combination, however, each polytope refers in general to a different PWA state-update and output function.

The discrete event system of the power system can be rewritten directly as inequality constraints (1c) by introducing binary and auxiliary continuous variables [3].

The above procedure yields an MLD system with two states, 302 z-variables, 31 δ-variables and 1660 inequality constraints. The derivation of the MLD system is performed by the compiler HYSDEL (HYbrid System DEscription Language) generating the matrices of the MLD system starting from a high-level description of the system [18].

4 Optimal Control Problem

4.1 Control Objectives

The control objectives are to bring V_{4m} as close to its reference value 1 as possible while fulfilling the soft constraints on the bus voltages $V_{2m} \in [0.95, 1.05]$, $V_{3m} \in [0.9, 1.1]$ and $V_{4m} \in [0.9, 1.1]$ and while switching the manipulated variables as little as possible.

The control moves can be classified as nominal and emergency control. During nominal control, the soft constraints on the bus voltages can be fulfilled by using only "cheap" control moves, i.e. capacitor bank switching s_C and changing the tap changer voltage reference $V_{4m,ref}$. If the soft constraints can't be met by only applying cheap control moves, the controller has to switch to emergency control and use the full range of available control moves including load-shedding which is considered to be very expensive.

4.2 MPC

As introduced in [19], *Model Predictive Control* (MPC) is well suited for finding control laws in an optimal way for hybrid systems described in the MLD framework. Here, an objective function is used that penalizes with the ∞-norm over a finite horizon the following three terms. (i) the deviation of the load voltage V_{4m} from its reference, (ii) the switching of the manipulated variables and (iii) the violation of the soft constraints. The control law is then obtained by minimizing the objective function subject to the mixed-integer linear inequality constraints of the MLD model (1) and the physical constraints on the manipulated variables. As we are using the ∞-norm, this minimization problem amounts to solving a *Mixed-*

Integer Linear Program (MILP). For details concerning the set up of the MPC formulation in connection with MLD models, the reader is referred to [3] and [2]. Details about MPC can be found in [14].

4.2.1 Cascaded Controller Scheme

The reference voltage $V_{4m,ref}$ is used to communicate to the internal controller of the tap changer whether the tap position shall be kept, increased or decreased by one step. In order to avoid sensitivity problems with this controller, we introduce the tapping strategy $\Delta n_T \in \{0, n_{step}, -n_{step}\}$, and we consider for the MPC formulation that Δn_T is manipulated directly subject to the limitation that it can only be changed every 30 s. Then for the actual implementation we need to convert back from Δn_T calculated by MPC to $V_{4m,ref}$ applied to the tapping controller.

$$V_{4m,ref}(t) = \begin{cases} V_{4m,ref}(t-1) & \text{if } \Delta n_T(t) = 0, \\ 0.8 & \text{if } \Delta n_T(t) = -n_{step}, \\ 1.2 & \text{if } \Delta n_T(t) = n_{step}. \end{cases}$$

Note that 0.8 and 1.2 constitute the minimal and maximal admissible values of $V_{4m,ref}$ and $n_{step} = 0.02$ is the physical step size of the tap changer.

4.2.2 MPC Objective Function

According to [2] and Section 4.1 and employing the cascaded controller scheme, the optimal control problem

$$\min_{\Delta u(0),...,\Delta u(N-1)} J := \sum_{k=0}^{N-1} \left(\|V_{4m}(t+k|t) - 1\| + \|Q\Delta u(k)\|_\infty \right)$$
$$+ \sum_{k=1}^{N-1} \left(\underline{S}(t+k|t) + \overline{S}(t+k|t) \right)$$

is considered subject to the evolution of the MLD model (1) over the prediction horizon N and additional physical constraints on the manipulated variables as defined in Section 2.1. The switching of the manipulated variables $\Delta u := [\Delta n_T, \Delta s_C, \Delta s_L]^T$ is weighted by the matrix $Q := \text{diag}(q_1, q_2, q_3)$. Note that due to the cascaded controller scheme, the tap changer strategy Δn_T is weighted rather than the reference voltage $V_{4m,ref}$. From a physical point of view this is reasonable, too, as the mechanical wear of the transformer results from tapping and not from changes in the reference voltage.

Having set the weight on the voltage deviation $\|V_{4m}(t+k|t) - 1\|$ to one, the weight Q on the manipulated variables is chosen such that control moves are performed when the penalty on the voltage deviation exceeds a certain limit. Therefore, Q is derived by comparing the cost on the voltage deviation $\|V_{4m}(t+k|t) - 1\|$ when refraining from using a control move with the cost of performing a control action [12]. Assume, that the tap changer may be moved by one step up or down, if this results in a reduction of the voltage deviation by at least 0.004.

As one step is given by $n_{step} = 0.02$, we get $q_1 = \frac{0.004}{0.02} = 0.2$. Analogously, assuming that one part of the capacitor bank may be switched if this reduces the voltage deviation by 0.03 or more leads to $q_2 = \frac{0.03}{1} = 0.03$. As the lower and upper voltage constraints on V_{4m} are equal to 0.9 and 1.1, respectively, the maximal cost resulting from the voltage deviation is 0.1. Load-shedding has to be avoided unless a soft constraint on the bus voltages is violated. This implies $q_3 \gg \frac{0.1}{1} = 0.1$ and therefore, $q_3 = 1$ is chosen.

In order to take the soft constraints on the lower bounds into account, the binary variables $\underline{\delta}_i, i \in \{2, 3, 4\}$ indicating the violation of the i-th lower soft constraint are introduced:

$$[\underline{\delta}_i(t) = 1] \leftrightarrow [V_{im}(t) < \underline{V}_{im}],$$

where $\underline{V}_{2m} = 0.95$, $\underline{V}_{3m} = \underline{V}_{4m} = 0.9$ are the lower bounds on the respective bus voltages as defined in Section 4.1. Additionally, the slack variables $\underline{s}_i \geq 0, i \in \{2, 3, 4\}$ denote the degree of the violation:

$$\underline{s}_i(t) = \begin{cases} 0 & \text{if } \underline{\delta}_i(t) = 0, \\ \underline{V}_{im} - V_{im}(t) & \text{if } \underline{\delta}_i(t) = 1. \end{cases}$$

Penalizing the violation as well as the degree of the violation yields

$$\underline{S}(t+k|t) = p \cdot \sum_{i=2}^{4} \left(\underline{\delta}_i(t+k|t) + \underline{s}_i(t+k|t) \right).$$

The soft constraints on the upper bounds $\overline{S}(t + k|t)$ are penalized accordingly. The weight p is chosen such that the full range of control moves can be used to remove the violation. This reasoning leads to $p \gg \|q \cdot \Delta u_{max}\|_\infty = 3$ and to the choice $p = 10$.

This choice of the penalties will cause emergency actions to be triggered as soon as one of the soft constraints is violated and will avoid the soft constraints to be violated unless absolutely necessary.

4.3 Control Experiments

This section presents control experiments using the cascaded controller scheme. MPC employs the derived MLD model as prediction model, whereas the "real" power system is described by the more accurate nonlinear model [7, 11]. As the load states and the tap changer position can be easily measured or estimated, we assume that they are available for the MPC controller. Given the sampling time $T_s = 30\,$s, the length of the prediction horizon N is set to 3, making sure that the prediction time interval amounts to $90\,$s and thus exceeds the dominant time constant $T_p = T_q = 60\,$s of the load. Then, given the initial states and inputs of Section 2.2 and applying the fault in line L3 at time $t = 100\,$s, we obtain the following closed-loop results.

Figure 4 shows the bus voltages V_{3m} and V_{4m} of the original nonlinear model as well as the ones of the MLD model. V_{2m} is

not depicted, as it remains within $[1.02, 1.03]$ during the control experiment and is thus of minor interest. The manipulated variable $V_{4m,ref}$ together with its tolerance band is shown in Figure 5, whereas Figure 6 displays the trajectory of the tap changer position n_T. The manipulated integer variables are as follows: s_C is in the beginning equal to 2 and then set to 3 at time $t = 120\,$s, whereas s_L remains constant at 0.

The fault at time $t = 100\,$s reduces the bus voltages instantaneously leading to a voltage collapse if no appropriate control actions are taken as shown Figure 3. At the first sampling instant after the fault at time $t = 120\,$s, MPC predicts the potential collapse and connects the remaining part of the capac-
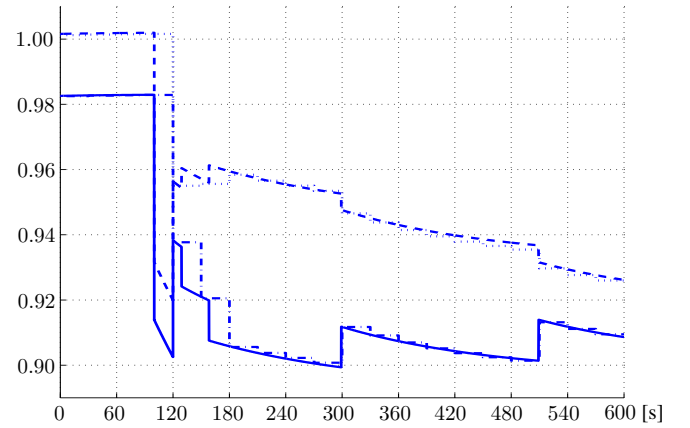

Figure 4: Bus voltages: V_{3m} of the nonlinear model: solid; V_{3m} of the MLD model: dash-dotted; V_{4m} of the nonlinear model: dashed; V_{4m} of the MLD model: dotted.
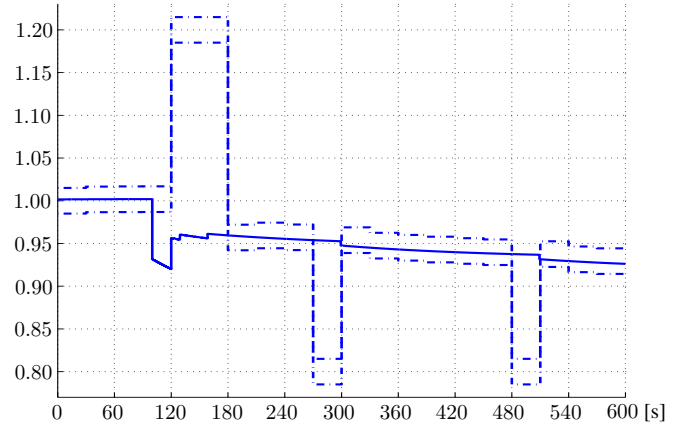

Figure 5: Voltages at bus 4: The tolerance band around $V_{4m,ref}$ is bounded by dash-dotted lines, V_{4m} is a straight line.
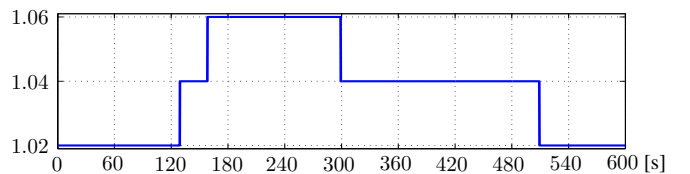

Figure 6: Tap changer position n_T.

itor bank to the power system by setting s_C to its maximum value 3. This control move is both necessary and sufficient to stabilize the system and therefore, thanks to the proper timing, only nominal control moves are needed to prevent a voltage collapse. Thus, load-shedding is avoided and s_L is kept at 0. Note that the proper timing of the control actions is important, as connecting the capacitor bank a few sampling instants later would lead to a severe violation of the lower soft constraint on V_{3m} that could not be removed by nominal control moves only (i.e. within the sampling time). As a result, emergency control would be applied and part of the load would be shed in order to meet the soft constraints.

Apart from the capacitor switching, $V_{4m,ref}$ is set to its maximum at time $t = 120$ s in order to step up the tap changer twice thus reducing the deviation of V_{4m} from its reference. At $t = 270$ s and $t = 480$ s, however, MPC issues tap down commands in order to avoid violations of the lower soft constraint on V_{3m}. According to the objective function and the horizon, this is the optimal sequence of control moves. If desired, a different tuning of the objective function and a longer prediction horizon can avoid the tap up and down actions and keep the tap changer at its initial position.

In Figure 4, the modelling error which results from approximating the nonlinear DAE by PWA functions can be seen as a small mismatch between the respective bus voltages of the nonlinear and the MLD model. Increasing the number of partitions of the PWA approximation would reduce the errors arbitrarily. The major error, however, is introduced by discretizing the time with $T_s = 30$ s. Using a sampling time of 10 s reduces this error significantly. For $T_s = 10$ s and $N = 7$, similar results are obtained with the only difference that the controller issues the first stabilizing control moves already at time $t = 110$ s. Furthermore, the system response of the MLD prediction model is smoother.

The computation times for solving the optimal control problem at each time-step when running CPLEX 8.0 on a Pentium IV 2.8 GHz machine are as follows. For $T_s = 30$ s and a horizon $N = 3$, the computation time is on average 1.8 s and always less than 3.5 s. For $T_s = 10$ s and $N = 7$, the respective times are 11 s and 160 s.

5 Conclusions

The example power system [7, 11] was modelled as an MLD system. Replacing the nonlinearities by PWA approximations introduces only small modelling errors hardly manifesting themselves in the control experiments thus proving the usefulness of the MLD modelling approach. We have shown that the load voltage V_{4m} can be stabilized by an appropriately tuned MPC controller using only nominal control moves. The tuning of the controller is straightforward and systematic allowing us to easily distinguish between nominal and emergency control moves.

Future research will be directed to avoid online optimization by

deriving the explicit feedback law [1, 4], to show closed-loop stability and to extend the size of the example power system.

References

[1] A. Bemporad, F. Borrelli, and M. Morari. Optimal controllers for hybrid systems: Stability and piecewise linear explicit form. *Proceedings of 39th Conference on Decision and Control*, 2000.

[2] A. Bemporad, F. Borrelli, and M. Morari. Piecewise linear optimal controllers for hybrid systems. In *Proceedings American Control Conference*, pages 1190–1194, 2000.

[3] A. Bemporad and M. Morari. Control of Systems Integrating Logic, Dynamics, and Constraints. *Automatica*, 35(3):407–427, March 1999.

[4] A. Bemporad, M. Morari, V. Dua, and E. N. Pistikopoulos. The explicit linear quadratic regulator for constrained systems. *Automatica*, 38(1):3–20, 2002.

[5] G. Ferrari-Trecate, M. Muselli, D. Liberati, and M. Morari. A clustering technique for the identification of piecewise affine systems. *Automatica*, 39(2):205–217, 2003.

[6] CIGRE Task Force. System protection schemes in power networks. Technical report, 2000.

[7] T. Geyer, M. Larsson, and M. Morari. Hybrid control of voltage collapse in power systems. Technical report, AUT02-12, Automatic Control Laboratory, ETH Zürich, Switzerland, July 2002.

[8] W.P.M.H. Heemels, B. De Schutter, and A. Bemporad. Equivalence of hybrid dynamical models. *Automatica*, 37(7):1085–1093, July 2001.

[9] P. Julian. *A Toolbox for the Piecewise Linear Approximation of Multidimensional Functions*, July 2000.

[10] P. Kundur. *Power System Stability and Control*. Power System Engineering Series. McGraw-Hill, New York, 1994.

[11] M. Larsson. The ABB Power Transmission Test Case. Technical report, ABB Corporate Research, www.dii.unisi.it/~hybrid/cc/, February 2002.

[12] M. Larsson, D. J. Hill, and G. Olsson. Emergency voltage control using search and predictive control. *International Journal of Power and Energy Systems.*, 24(2):121–130, 2002.

[13] M. Larsson and D. Karlsson. System protection scheme against voltage collapse based on heuristic search and predictive control. *IEEE Transactions of Power Systems*, to appear.

[14] J.M. Maciejowski. *Predictive Control*. Prentice Hall, 2002.

[15] C. Moors, D. Lefebvre, and T. van Cutsem. Design of load shedding schemes against voltage instability. In Power Engineering Society Winter Meeting, vol. 2, pp. 1495-1500, 2000.

[16] C. Rehtanz and J. Bertsch. A new wide area protection system. In *Bulk Power System Dynamics and Control*, 2001.

[17] C. W. Taylor. *Power System Voltage Stability*. McGraw-Hill, 1994.

[18] F.D. Torrisi and A. Bemporad. HYSDEL — A Tool for Generating Computational Hybrid Models for Analysis and Synthesis Problems. *IEEE Transactions on Control System Technology*, to appear.

[19] M. L. Tyler and M. Morari. Propositional Logic in Control and Monitoring Problems. *Automatica*, 35(4):565–582, April 1999.

[20] T. van Cutsem and C. Vournas. *Voltage Stability of Electric Power Systems*. Power Electronics and Power Systems Series. Kluwer Academic Publishers, 1998.